



1995-96 KIRIS OPEN-RESPONSE ITEM SCORING WORKSHEET

Grade 11 — Mathematics Question 6

The academic expectations addressed by this item include:

1.5 - 1.9 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

2.11 Students understand mathematical change concepts and use them appropriately and accurately.

The core content assessed by this item includes:

Algebraic Ideas Concept

- Students should understand variables and constants in expressions, equations, and inequalities.

Skill

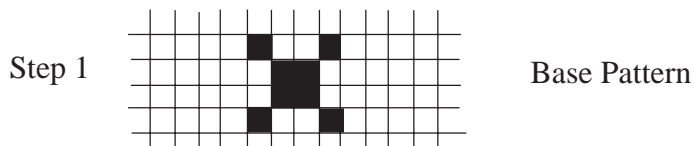
- Students should be able to represent patterns using functions.

Number/Computation Concept

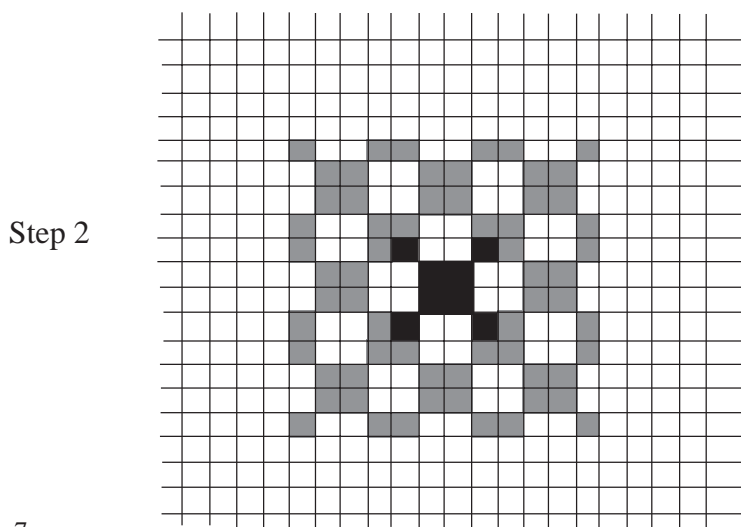
- Students should understand sequences, series (e.g., types, limits, terms).

6. Geometric Floor Pattern

Ms. Martine's class chose a base pattern to be used on the classroom floor. The figure below illustrates the pattern they chose. The base pattern is a large square made up of 8 unshaded and 8 shaded unit squares.



Then they added duplicates of the pattern to the original base pattern to create Step 2.



Question 1: Geometric Floor Pattern Continued

- How many duplicates of the base pattern were added in order to complete **Step 2**? How many shaded unit squares were **added** in **Step 2**?
- Each step is accomplished by surrounding the existing figure with copies of the base pattern. How many duplicates of the base pattern would you add to complete **Step 3**? How many shaded unit squares would you need to add to complete **Step 3**?
- Using the information from **Steps 1, 2, and 3**, how many shaded unit squares will be added in **Step 6**?
- Write a generalization, or rule, for determining the number of shaded unit squares added in **Step n** . Explain how you determined this generalization.

BE SURE TO LABEL YOUR RESPONSES (a), (b), (c), AND (d).

SCORING GUIDE

Score	Description														
4	<p>Response shows correct numbers of base patterns and shaded squares added for both Steps 2 and 3 and shows establishment of a pattern to find the number of shaded squares added in Step 6. Response includes a generalization for Step n with an explanation of how the generalization was determined</p> <table> <tr> <td>base patterns added</td><td>shaded squares added</td></tr> <tr> <td>*step 2 8 (1x8)</td><td>$8 \times 6 = 64$ $1 \times 8 \times 8$</td></tr> <tr> <td>step 3 16 (2x8)</td><td>$16 \times 8 = 128$ $2 \times 8 \times 8$</td></tr> <tr> <td>step 4 24 (3x8)</td><td>$24 \times 8 = 192$ $3 \times 8 \times 8$</td></tr> <tr> <td>step 5 32 (4x8)</td><td>$32 \times 8 = 256$ $4 \times 8 \times 8$</td></tr> <tr> <td>step 6 40 (5x8)</td><td>$40 \times 8 = 320$ $5 \times 8 \times 8$</td></tr> <tr> <td>step n $[(n-1) \times 8]$</td><td>$(n - 1) \times 8 \times 8$</td></tr> </table>	base patterns added	shaded squares added	*step 2 8 (1x8)	$8 \times 6 = 64$ $1 \times 8 \times 8$	step 3 16 (2x8)	$16 \times 8 = 128$ $2 \times 8 \times 8$	step 4 24 (3x8)	$24 \times 8 = 192$ $3 \times 8 \times 8$	step 5 32 (4x8)	$32 \times 8 = 256$ $4 \times 8 \times 8$	step 6 40 (5x8)	$40 \times 8 = 320$ $5 \times 8 \times 8$	step n $[(n-1) \times 8]$	$(n - 1) \times 8 \times 8$
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3	<p>Response shows correct number of base patterns and shaded squares added for Steps 2 and 3 and shows establishment of a correct pattern to find the number of shaded squares added in Step 6. Student attempts to generalize for Step n.</p> <p>OR</p> <p>Response shows a clear understanding of all parts of the task, but has incorrect answer(s) due to a computational error.</p>														
2	<p>Response shows correct numbers of base patterns and shaded squares added in Step 2 and 3 with remaining parts incorrect or not attempted.</p> <p>OR</p> <p>Response clearly shows understanding of pattern expansion with or without some calculation error.</p>														
1	Response shows correct number of base patterns and/or shaded squares added for Step 2 only.														
0	Response is incorrect or irrelevant.														
Blank	Blank/no response.														



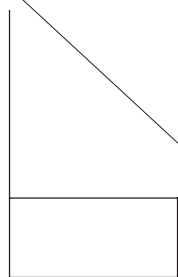
KIRIS ASSESSMENT ANNOTATED RESPONSE GRADE 11 MATHEMATICS

Sample 4-Point Response of Student Work

Student addresses the dual requirements (base patterns and shaded unit squares) of parts a and b correctly.

Student points out that the algebraic representation for step n does not include $n = 1$.

- A) 8 base patterns were added to complete step 2. 64 shaded units squares were add to complete step 2.
- B) 16 base units would have to be added to complete step three. 128 shaded units squares would have to be added to complete step 3.
- C) 320 shaded units will be added to complete step 6.
- D) $(n-1) 8 (8) =$ shaded units squares in step n . Work for all except step $\textcircled{1}$. I determined this by following the patterns of 8 Bases + 8 Bases = the next amount of Bases.



$\textcircled{1}$	$\textcircled{2}$	$\textcircled{3}$	$\textcircled{4}$	$\textcircled{5}$	$\textcircled{6}$
1	$\textcircled{8}$	16	24	32	40
$\textcircled{8}$	64	128	192	256	320

Student extends understanding of pattern to calculate the number of shaded unit squares added in step 6.

The rule for generalizing the pattern is supported by a demonstration of how the rule was derived and an explanation for determining the number of bases.

Student constructs an algebraic representation of the shaded unit squares using the variable n to represent the step index, the n th term of the sequence.



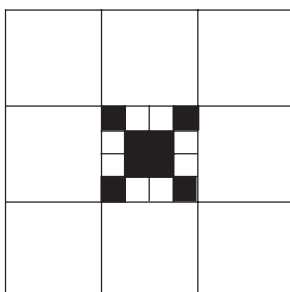
KIRIS ASSESSMENT ANNOTATED RESPONSE GRADE 11 MATHEMATICS

Sample 3-Point Response of Student Work

Student presents clear, correct answers for parts a and b. Simple drawing supports work.

- a) 8 base patterns were added.
 $8 \times 8 = 64$ shaded unit squares were added.
- b) 16 base patterns were added
 $16 \times 8 = 128$ shaded unit squares were added.
- c) 24 - base patterns in Step 4
32 - base patterns in Step 5
40 - base patterns in Step 6
 $40 \times 8 = 320$ shaded unit squares in Step 6

- d) $(n-1)8$
using the information that I already have. I proved this formula to be correct.



Note: Student used grid in Student Response Book to organize and simplify the drawing.

The number of base patterns is correctly calculated for steps 4, 5, and 6, and then the student multiplies to determine the number of shaded unit squares in step 6.

The student merely gives the rule for determining the number of base patterns, rather than the number of shaded unit squares added in step n . Rule shows understanding of the concept but work is not carried to the required step, explanation is incomplete.



KIRIS ASSESSMENT ANNOTATED RESPONSE

GRADE 11 MATHEMATICS

Sample 2-Point Response of Student Work

The student gives correct answers to parts a and b.

The generalization for step n is incorrect and reflects an incomplete, simplistic analysis of the pattern.

- A) 8, 64
- B) 16, 128
- C) 1024
- D) Times By 8

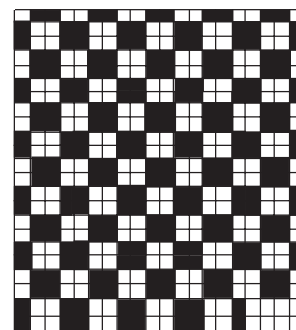
Lack of supporting work for part c makes analysis of reason for incorrect answer difficult to determine, although student appears to understand that correct answer is a multiple of 8.

Sample 1-Point Response of Student Work

The student correctly states that 8 duplicates of the pattern were added to complete step 2, but does not correctly describe the composition of the pattern after that step.

Ms. Martine's class chose a pattern to be used on the classroom floor. The base illustrates the pattern they chose. The last pattern is a large square made up of 8 white and 8 shaded unit squares. Eight duplicates of the base pattern were added in order to complete the pattern in step 2. 10 more patterns were added in step 3 and this is how the floor will look when it is finished.

GRAPH



Note: Student used the grid in the student response book to extend the pattern to “finish” tiling the given grid.

INSTRUCTIONAL STRATEGIES

Geometric Floor Plan

Study patterns and models of patterns, patterns that are geometric and arithmetic.

Study a pattern and write an equation. Take it further than just doing problems with patterns, explore sequences and series and have students create equations for the terms and test their equations.

Create organized listings of the steps in pattern development, generalize the change(s) occurring, examine rules and the generation of rules to represent the changes.

Use KIRIS-like open response questions in classroom instruction and assessment. Model strategies for explaining work to fellow mathematicians. Model and have students develop and use scoring guides with open response items. Encourage students to explore highlighting and underlining strategies as organizers, stressing that only evidence found in Student Response Book is scored on KIRIS open responses.

Infuse lessons with the use of a variety of instructional approaches and strategies:

- use mathematical tools, manipulatives, hands on activities, cooperative group work, higher order thinking skills, video tapes, multiple intelligences approaches, mappings, graphic organizers, etc.

Explore appropriate use of calculators, both as tools and instruments for checking work.

REFERENCES

TRANSFORMATIONS Kentucky's Curriculum Framework

Academic Expectations 1.5-1.9 and 2.7 through 2.13

KDE's Core Content for Assessment

Mathematics, examine curriculum alignment from P through 12

KDE's web site at <http://www.kde.state.ky.us>

explore curriculum pages, examine units of study, etc.

Curriculum and Evaluation Standards for School Mathematics,

Professional Standards for Teaching Mathematics, Assessment

Standards for School Mathematics, and Addenda Series from NCTM.

Telephone: 703-620-9840, web site at <http://www.nctm.org>